Coherence of Low-frequency Sound Signals Propagating through a Fluctuating Ocean: Analysis and Theoretical Interpretation of 2004 NPAL Experimental Data

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LONG-TERM GOALS

To theoretically study low-frequency, long-range sound propagation in a fluctuating ocean, including studies of 3D effects.

To compare obtained theoretical results with experimental data.

OBJECTIVES

To finalize development of a new, modal, 3D theory of low-frequency, long-range sound propagation through an ocean with random inhomogeneities which are statistically isotropic in a horizontal plane.

Based on this theory, to develop computer codes for calculating the first two statistical moments of a sound field propagating through the ocean perturbed by internal gravity waves.

To continue comparison between theoretical predictions and data obtained during the 1998-1999 and 2004 NPAL experiments.

APPROACH

Studies of the mean field and the coherence function of low-frequency sound waves propagating over long-ranges in a fluctuating ocean are important for many applications, including source detection and ranging. These studies have been done in several papers using different approaches (see [1] for the literature review). For low-frequency, long-range sound propagation, a modal approach for calculation of the mean field and the coherence function seems to be the most adequate. This approach has been used in a number of papers both for 2D and 3D geometries. For the 3D case, most of the equations obtained in the literature for the coherence function are too involved to be solved numerically, e.g. see [1-4].

To make numerical calculations of the coherence function feasible, in this project we assume [11-13] that a sound source is omni-directional and random inhomogeneities are statistically homogeneous and

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14. ABSTRACT
To theoretically study low-frequency, long-range sound propagation in a fluctuating ocean, including studies of 3D effects. To compare obtained theoretical results with experimental data.

15. SUBJECT TERMS

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isotropic in a horizontal plane. Both these assumptions are valid with a good accuracy for many practical applications. With these assumptions, it is worthwhile to do analysis in a cylindrical coordinate system and to decompose the sound field into a sum of acoustic modes and azimuthal harmonics. Closed differential equations for the mean field and the coherence function of a monochromatic sound wave propagating in a random ocean are derived with the Chernov method. Then, explicit expressions for the mean field and the coherence function are obtained with the matrix Rytov method. The dimension of the scattering matrix appearing in the differential equation and in the explicit expression for the coherence function is much smaller than those in the previous works which makes numerical calculations of the coherence function feasible. Based on these theoretical results, the computer codes for calculations of the mean field and the coherence function are developed.

Dr. A. Voronovich and Dr. V. Ostashev are PI and Co-PI in this project.

WORK COMPLETED

During the reporting period, the following two tasks were completed:

Task 1. An explicit expression for the mean field of a sound wave propagating in a fluctuating ocean was derived with the Chernov and matrix Rytov methods. The computer codes were developed and used in numerical studies of the mean field.

Task 2. A closed differential equation for the coherence function was derived with the Chernov method. Then, an approximate, explicit expression for the coherence function was obtained with the matrix Rytov method. The dependence of the coherence function on different parameters of the problem was studied numerically with computer codes developed.

RESULTS

The following results were obtained in FY08:

Task 1.
An explicit expression for the mean field of a sound wave propagating in an oceanic waveguide with random inhomogeneities was derived using the Chernov and matrix Rytov methods. According to this expression, the mead sound field is a sum of acoustic modes whose amplitudes decrease exponentially with increasing propagation range R. The extinction coefficients of the mode amplitudes, s_{nn} are expressed in terms of the spectrum of IW fluctuations. (Here, the subscript n indicates the mode number). It was shown that the mean sound field calculated with the Chernov and matrix Rytov method agrees with that calculated in [5] using a diagram technique, which is a more general approach. This justifies the use of the Chernov and matrix Rytov methods in calculations of the mean field and the coherence function.

Numerical codes were developed for calculations of the extinction coefficients s_{nn} and the mean sound field in an oceanic waveguide perturbed by IWs with the Garrett-Munk spectrum. Figure 1 from Ref. [6] shows the inverse of the extinction coefficient, $1/s_{nn}$, which is the extinction length of the mode amplitude, versus the mode number n. In calculations, the sound frequency was $f = 40\,\text{Hz}$, the ocean depth $H = 5$ km, and the vertical profiles of the sound speed $c(z)$ and the Brunt-Väisälä frequency

$N(z)$ corresponded to those for transmission from the T1000 station in the LOAPEX experiment to "site R" near Hawaii [8]. Note that, for this geometry, the number of waterborne acoustic modes is 48. It follows from the Fig. 1 that the extinction length $1/s_{nn}$ increases with increasing mode number and can be as large as about 400 km for the waterborne modes. This implies that, for low-frequency sound waves, the mean field can persist upto propagation ranges of many hundreds of kilometers which can be important in applications [8].
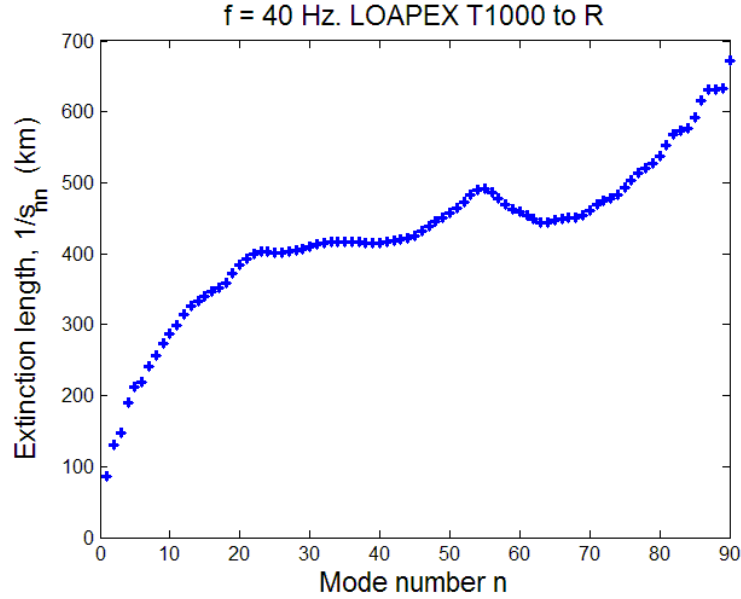

Figure 1. Extinction length of the mode amplitude versus the mode number. Sound frequency f = 40 Hz. The ocean stratification corresponds to that for transmission from the LOAPEX T1000 station to "site R" near Hawaii.
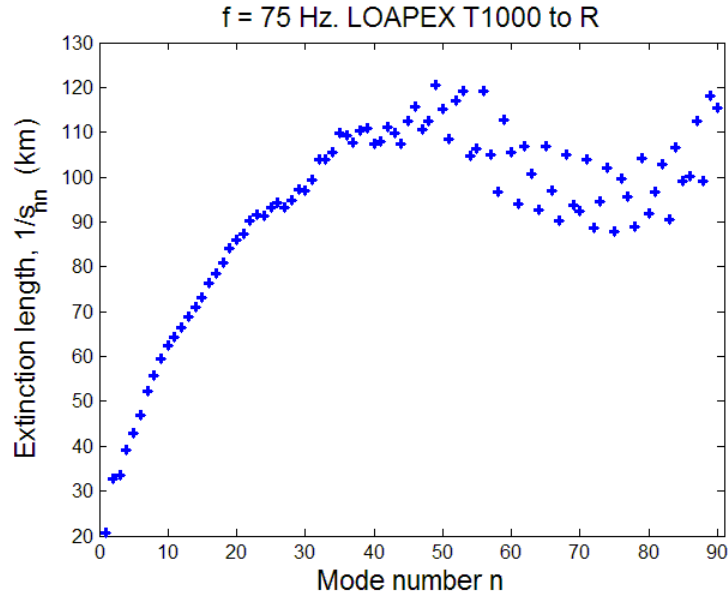

Figure 2. The same as in Fig. 1 but for the sound frequency f = 75 Hz.

Figure 2 is similar to Fig. 1 except that the extinction lengths are calculated for the sound frequency $f = 75\,\text{Hz}$, which was one the transmission frequencies in the LOAPEX experiment [7]. In comparison with Fig. 1, all acoustic modes shown in Fig. 2 are waterborne. Furthermore, in Fig. 2 the extinction lengths of the mode amplitudes increase with increasing mode number only upto $n \sim 50$. It also follows from Fig. 2 that the maximum values of the extinction lengths are of order 120 km and are smaller than those for $f = 40\,\text{Hz}$.
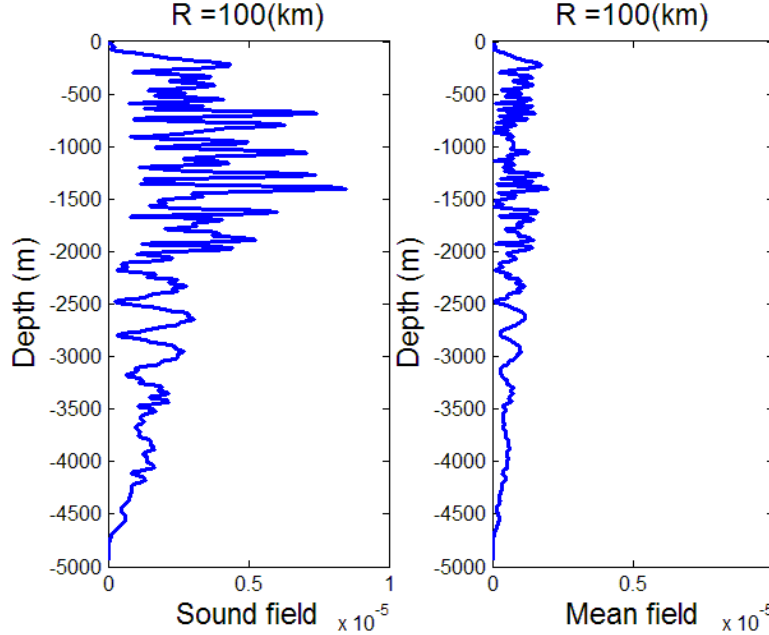

Figure 3. (Left) Sound field versus depth in the ocean without IWs . (Right) Mean sound field versus depth in a fluctuating ocean with IWs. For both plots, the ocean stratification is the same as that in Figs. 1 and 2, the sound frequency f = 75 Hz, and the propagation range R = 100 km.

Figure 3 compares the sound field in the ocean without IWs with the mean sound field in the presence of the IW field with the Garrett-Munk spectrum. When plotting the figure, it was assumed that the ocean stratification is the same as that in Figs. 1 and 2, the sound frequency $f = 75\,\text{Hz}$, the propagation range $R = 100$ km, and the source depth $z_s = -807$ m. It follows from Fig. 3 that the maximum amplitude of the mean sound field is only several times less than that of the sound field without IWs. Therefore, the mean sound field can probably be measured at the range $R = 100$ km. However, at $R = 588$ km (the distance between the LOAPEX T1000 station and "site R") the maximum amplitude of the mean sound field is less than that in the absence of IWs by two orders of magnitude which makes measurements of the mean field difficult.

Task 2.
A closed differential equation for the coherence function of a sound wave propagating in a fluctuating ocean was derived using the Chernov method. Then, this equation was approximately solved with the matrix Rytov method. As a result, an explicit expression for the coherence function was obtained. Both the differential equation and the explicit expression contain an interaction matrix which is expressed in terms of the spectrum of random inhomogeneities. Computer codes were developed for solving the

differential equation and for calculating the coherence using the explicit expression. Calculations of the coherence function are much faster with the explicit expression rather than with the differential equation while the difference between these two approaches is only of the order of a few percent. The numerical codes developed enable detailed analysis of the dependence of the coherence function on different parameters of the problem, e.g. frequency, range, hydrophone depth, etc. Some of the numerical results obtained in this analysis are presented below.
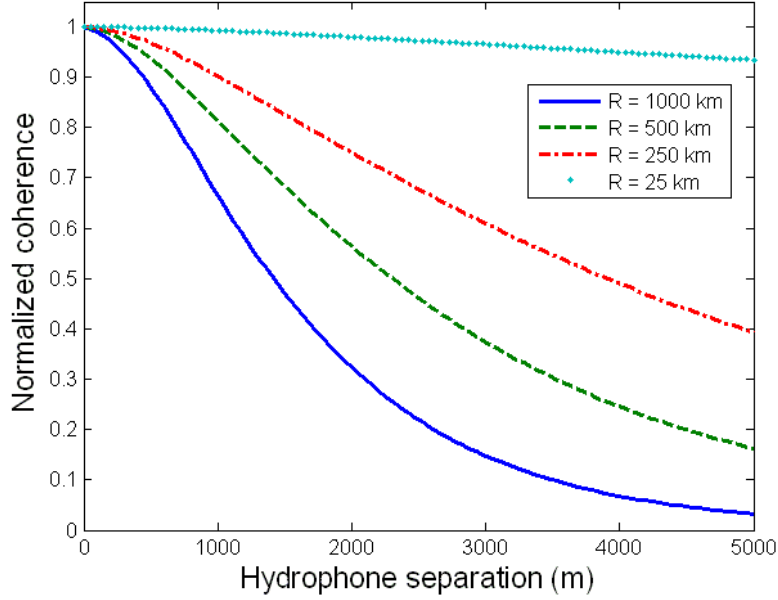

***Figure 4. The magnitude of the normalized coherence function versus
the horizontal hydrophone separation y for four propagation ranges R.***

In Fig. 4, the dependence of the magnitude of the depth-averaged coherence function (normalized by its maximum value) on the horizontal hydrophone separation y is plotted for four propagation ranges R, the sound frequency $f = 75\,\text{Hz}$, and the source depth $z_s = -807\,\text{m}$. At the range $R = 25\,\text{km}$, the coherence depends only slightly on y. An increase in the propagation range results in decreasing coherence, as it should be. It follows from Fig. 3, that, for the ranges $R = 500$ and $R = 1000$ km, the coherence radii of the sound field are 3 and 1.8 km, respectively.

The magnitude of the normalized coherence function versus the horizontal separation y is plotted in Fig. 5 for the propagation range $R = 3900$ km, the sound frequency $f = 75$ Hz, and the source depth $z_s = -807\,\text{m}$. The solid line is the theoretical result obtained with the numerical codes developed. Symbols are the values of the coherence calculated in Ref. [9] using experimental data obtained during the 1998-1999 NPAL experiment [10]. These values were obtained for two different time moments of the experiment (year days 285.2873 and 257.2873 of the experiment) and thus correspond to "realizations" of the coherence function. The overall agreement between the theory and experimental data in Fig. 5 is fair.

The dependence of the coherence function on y is often approximated as $\exp(-Ay^{\alpha})$, where A and α are coefficients. Numerical calculations with the code CAFI indicate that $\alpha = 1.5$. The dependence of the theoretical coherence in Fig. 5 on y can be divided into two regions separated by the red dashed line. In the first region, where $0 \le y \le 750\,\text{m}$, the coefficient $\alpha = 1.98$. In the second region, where $750 \le y \le 2500\,\text{m}$, this coefficient $\alpha = 1.43$ which is close to that predicted with CAFI.

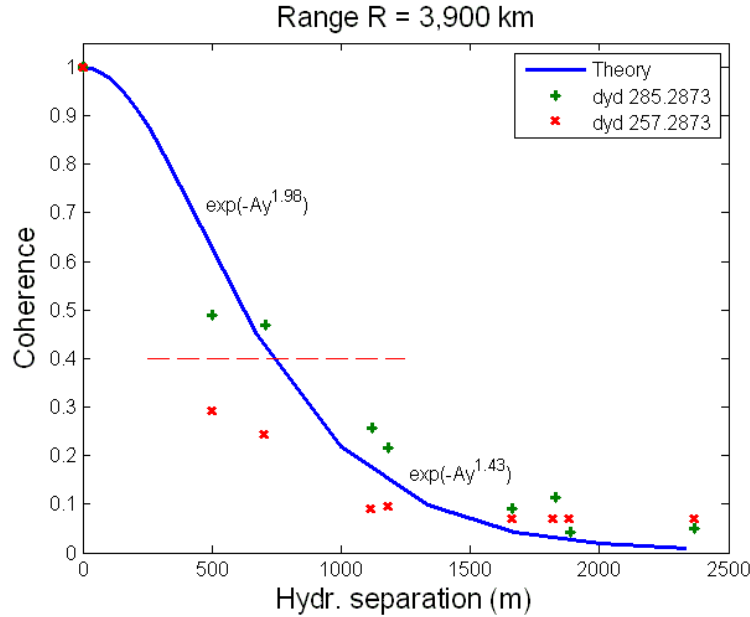*Figure 5. The magnitude of the normalized coherence function versus the horizontal hydrophone separation y. The solid line is the theoretical result. Symbols are the experimental data obtained during the 1998-1999 NPAL experiment.*

IMPACT/APPLICATIONS

Using the modal, 3D theory of sound propagation in a fluctuating ocean with horizontally isotropic statistics, closed differential equations and explicit expressions for the mean field and the coherence function due to a monochromatic, omni-directional source were obtained. The dependence of the mean field and the coherence function on parameters of the problem was studied numerically with the computer codes developed. The coherence function calculated theoretically agrees with experimental data obtained in the 1998-1999NPAL experiment.

RELATED PROEJCTS

1. "Multiple Scattering of Sound by Internal Waves and Acoustic Characterization of Internal Wave Fields in Deep and Shallow Water", ONR projects N00014-05-IP2-0024 and N00014-06-1-0010.

2. The 2004-2005 NPAL experiment, see Ref. [7].

3. The 1998-1999 NPAL experiment, see Ref. [10].

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11. A. G. Voronovich and V. E. Ostashev, "Coherence function of a sound field in an oceanic waveguide with horizontally isotropic random statistics", J. Acoust. Soc. Am. (2007) [in press].

12. A. G. Voronovich and V. E. Ostashev, "Application of the matrix Rytov method to the calculation of the coherence function of a sound field in an oceanic waveguide," J. Acoust. Soc. Am. **123**, No 5, Pt. 2, p. 3941 (2008) [published].

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